Atmospheres Notebook: Index

Physical Constants

(ref. Physics Today, Aug. 1995 pg. BG9
AU = astronomical unit = 1.4960 $\cdot 10^{13}$ cm
c = speed of light = $2.99792458 \cdot 10^{10}$ cm/s
$G = 6.67259(85) \cdot 10^{-8}$ dyne cm^2/gm^2
h = Planck's constant = $6.626075(40) \cdot 10^{-27}$ erg sec
k = Boltzmann's constant = 1.380658(12) $\cdot 10^{-16}$ erg/K
N_a = Avogadro's number = 6.0221367(36) $\cdot 10^{23}$ gm/mole
σ = Stefan Boltzmann constant = 5.67051(19) $\cdot 10^{-5}$ erg/s/cm^2/K^4
$S_0 = 1.37 \cdot 10^6$ erg/cm^2
$R_{univ} = N_a \cdot k = 8.3143 \cdot 10^7$ erg/mole/K
$\overline{\text{mw}}_{H_2O}$ = 18.016 gm/mole
$\overline{\text{mw}}_{O3}$ = 47.9982 gm/mole

Earth Constants

R_{eq} = 6378.388 km
$M_e = 5.975 \cdot 10^{27}$ gm
$\overline{\text{mw}}_{dryair}$ = 28.964 gm/mole
$R_g = 0.28705 \cdot 10^7$ erg/gm/K
N_0 = Loschmidt's number = 2.686763(23) $\cdot 10^{19}$ molecules/cm^3
g_s = Standard surface gravity = 980.665 cm/s
P_s = Standard surface pressure = 1.01325 $\cdot 10^6$ dyne/cm^2
c_p = 1.006 $\cdot 10^7$ erg/gm/K at stp
c_v = 0.718 $\cdot 10^7$ erg/gm/K at stp
$\gamma = c_p/c_v$ = 1.401 at stp

Unit Conversion

1 knot = 1 nautical mile/hour = 1.151555 mile/hour = 1.853248 Km/hr
1 megaton = $4.2 \cdot 10^{22}$ erg = $4.2 \cdot 10^{15}$ Joule
1 AU = $1.495 \cdot 10^8$ Km's = 8.31 light minutes

Planet Size Information

	area Km**2	volume Km**3	Mass grams	Mass Me	density gm/cm**3
Venus	4.6011e+08	9.2805e+11	4.8850e+27	0.817	5.264
Earth	5.1006e+08	1.0832e+12	5.9790e+27	1.000	5.520
Mars	1.4416e+08	1.6275e+11	6.4570e+26	0.108	3.967
Jupiter	6.1469e+10	1.4313e+15	1.8986e+30	317.551	1.327
Saturn	4.2694e+10	8.2713e+14	5.6854e+29	95.089	0.687
Uranus	8.0840e+09	6.8336e+13	8.6825e+28	14.522	1.271
Neptune	7.6280e+09	6.2637e+13	1.0310e+29	17.244	1.646
Titan	8.3323e+07	7.1519e+10	1.3500e+26	0.023	1.888

Planet Rotation Period

	hours	hr:mn:secs	day/earth	deg/day
Venus	-5833.2000000	-5833:12: 0.0	-243.05000	-1.481177
Earth	23.9333330	23:56: 0.0	0.99722	361.002791
Mars	24.6333330	24:38: 0.0	1.02639	350.744254
Jupiter: SI..	9.8416675	9:50:30.0	0.41007	877.900010
Jupiter: SII.	9.9279533	9:55:40.6	0.41366	870.270008
Jupiter: SIII	9.9249197	9:55:29.7	0.41354	870.536010
Saturn: SI..	10.2333290	10:14: 0.0	0.42639	844.300032
Saturn: SIII	10.6562220	10:39:22.4	0.44401	810.793919
Uranus	-17.2333330	-17:14: 0.0	-0.71806	-501.353975
Neptune	16.1100000	16: 6:36.0	0.67125	536.312849
Titan	382.6799927	382:40:48.0	15.94500	22.577611

Orbital Parameters

	a AU	period years	inclin deg	a.node deg	arg.perih deg	ecc	L(6430) deg
Venus	0.7233	0.62	3.39	76.55	131.25	0.006818	267.689
Earth	1.0000	1.00	0.00	0.00	102.77	0.016704	99.372
Mars	1.5237	1.88	1.85	49.45	335.82	0.093329	195.082
Jupiter	5.2025	11.87	1.31	100.35	15.46	0.048075	329.233
Saturn	9.5531	29.53	2.49	113.55	92.20	0.051565	238.772
Uranus	19.2642	84.55	0.77	73.99	175.28	0.046227	253.217
Neptune	30.2337	166.24	1.77	131.82	7.60	0.007972	274.526
Titan	0.0082	0.04	0.33	0.00	0.00	0.030000	0.000

Planet Pole

	<r> AU	inclin deg	RA deg	DEC deg
Venus	0.7233	177.40	272.78	67.21
Earth	1.0000	23.40	0.00	90.00
Mars	1.5237	25.20	317.57	57.52
Jupiter	5.2025	3.10	268.04	64.49
Saturn	9.5531	26.70	39.36	83.45
Uranus	19.2642	97.90	257.43	-15.10
Neptune	30.2337	29.00	295.33	40.65
Titan	0.0082	0.00	0.00	0.00

Velocity

	V_orbit Km/s	V_escape Km/s	V_equator Km/s	V_sound Km/s
Venus	35.022	10.378	-0.002	0.236
Earth	29.786	11.192	0.465	0.320
Mars	24.130	5.045	0.240	0.227
Jupiter	13.059	60.511	12.572	0.816
Saturn	9.637	36.376	9.871	0.716
Uranus	6.786	21.411	-2.589	0.543
Neptune	5.417	23.670	2.689	0.543
Titan	5.573	2.645	0.012	0.185

Effective (Teff,Peff) Values

	Teff K	Peff mBar	rho gm/cm^3	H Km	V_sound m/s	tau_rad years	phase deg
Venus	229.0	20.000	4.6230e-05	4.862	235.8	0.007	3.94
Earth	255.0	500.000	6.8297e-04	7.471	320.2	0.131	39.47
Mars	210.0	6.000	1.5124e-05	10.605	226.6	0.006	1.17
Jupiter	124.0	400.000	8.6132e-05	19.146	816.3	4.534	67.39
Saturn	95.0	300.000	8.1128e-05	36.975	715.5	20.794	77.27
Uranus	59.0	430.000	2.0161e-04	24.234	543.4	131.311	84.15
Neptune	59.0	500.000	2.3443e-04	19.206	543.4	121.008	77.67
Titan	85.0	1.000	4.0568e-06	18.152	184.7	0.053	82.54

Stratopause Values

	T	P	rho	H	V_sound	tau_rad	phase
	K	mBar	gm/cm^3	Km	m/s	years	deg
Venus	400.0	50.000	6.6166e-05	8.492	311.7	0.003	1.85
Earth	270.0	1.000	1.2901e-06	7.911	329.5	0.000	0.08
Mars	140.0	0.005	1.8905e-08	7.070	185.0	0.000	0.00
Jupiter	163.0	3.000	4.9143e-07	25.167	935.9	0.015	0.45
Saturn	145.0	2.000	3.5435e-07	56.435	884.0	0.039	0.48
Uranus	120.0	1.000	2.3053e-07	49.290	775.0	0.036	0.15
Neptune	130.0	2.000	4.2559e-07	42.319	806.7	0.045	0.10
Titan	170.0	1.000	2.0284e-06	36.304	261.3	0.007	43.69

Tropopause Values

	T	P	rho	H	V_sound	tau_rad	phase
	K	mBar	gm/cm^3	Km	m/s	years	deg
Venus	250.0	100.000	2.1173e-04	5.307	246.4	0.026	14.83
Earth	217.0	100.000	1.6051e-04	6.358	295.4	0.043	14.96
Mars	140.0	0.010	3.7809e-08	7.070	185.0	0.000	0.01
Jupiter	110.0	140.000	3.3983e-05	16.984	768.8	2.273	50.28
Saturn	85.0	100.000	3.0224e-05	33.083	676.8	9.677	64.10
Uranus	53.0	110.000	5.7414e-05	21.770	515.1	46.340	73.81
Neptune	54.0	200.000	1.0246e-04	17.579	519.9	63.132	67.26
Titan	70.0	100.000	4.9261e-04	14.949	167.7	9.506	89.96

1 BAR Values

	T	P	rho	H	V_sound	tau_rad	phase
	K	BAR	gm/cm^3	Km	m/s	years	deg
Venus	360.0	1.000	1.4704e-03	7.643	295.7	0.087	41.57
Earth	288.0	1.000	1.2094e-03	8.438	340.3	0.182	48.82
Mars	140.0	0.007	2.6466e-05	7.070	185.0	0.024	4.59
Jupiter	165.0	1.000	1.6182e-04	25.476	941.6	4.811	68.57
Saturn	134.0	1.000	1.9172e-04	52.154	849.8	24.699	79.23
Uranus	76.0	1.000	3.6399e-04	31.217	616.8	142.873	84.62
Neptune	76.0	1.000	3.6399e-04	24.740	616.8	113.230	76.85
Titan	86.0	1.000	4.0096e-03	18.66	185.8	51.265	89.99

Surface (H2O) Values

	T	P	rho	H	V_sound	tau_rad	phase
	K	BAR	gm/cm^3	Km	m/s	years	deg
Venus	731.0	92.000	6.6619e-02	15.519	421.4	0.954	84.14
Earth	288.0	1.013	1.2252e-03	8.438	340.3	0.184	49.19
Mars	214.0	0.007	1.7314e-05	10.807	228.7	0.007	1.29
Jupiter	294.0	7.000	6.3574e-04	45.394	1256.9	5.953	72.40
Saturn	313.0	21.000	1.7237e-03	121.823	1298.8	40.698	83.41
Uranus	366.0	260.000	1.9651e-02	150.336	1353.5	332.598	87.68
Neptune	360.0	283.000	2.1746e-02	117.192	1342.3	301.495	84.98
Titan	94.0	1.500	5.5026e-03	20.074	194.3	58.887	89.99

Adiabatic Lapse Rate, Dry

from the first law of thermodynamics

$$dQ = dU + \delta W = n \cdot c_v dT + PdV = 0$$

where c_v is given in units of erg/K/mole and n is the number of moles. The derivative of the ideal gas law, $P \cdot V = nRT$, is

$$VdP + PdV = nRdT$$

equating PdV and noting that $R = c_p - c_v$ yields

$$dQ = nc_v dT - VdP + n(c_p - c_v)dT$$

$$dQ = nc_p dT - VdP = 0 \quad \text{for adiabatic}$$

$C_p = c_p/\overline{\text{mw}}$ and $\rho = n \cdot \overline{\text{mw}}/V$ so that

$$\frac{dT}{dP} = \frac{V}{n \cdot c_p} = \frac{1}{C_p \rho}$$

From hydrostatic equilibrium and the gas law we can convert from pressure to height coordinates:

$$dP = -g\rho dz$$

$$\left.\frac{dT}{dz}\right|_a = -\frac{g}{C_p}$$

$$\Gamma_a \equiv -\left.\frac{dT}{dz}\right|_a = \frac{g}{C_p}$$

	<g>	Cp	Adiabatic Lapse Rate	Rg	Autoconvective Lapse Rate
	cm/s**2	J/gm/K	K/Km	J/gm/K	K/Km
Venus	889.89	0.8501	10.468	0.18892	47.104
Earth	979.86	1.0040	9.760	0.28710	34.130
Mars	374.10	0.8312	4.500	0.18892	19.802
Jupiter	2425.61	12.3591	1.963	3.74518	6.477
Saturn	1000.09	14.0129	0.714	3.89246	2.569
Uranus	880.07	13.0137	0.676	3.61491	2.435
Neptune	1110.46	13.0137	0.853	3.61491	3.072
Titan	135.80	1.0440	1.301	0.29000	4.683

Autoconvective Lapse Rate

From hydrostatic equilibrium and the ideal gas law, $P = R_g \rho(z) T(z)$ we have

$$\rho = -\frac{1}{g}\frac{dP}{dz} = -\frac{R_g}{g}\left[T(z)\frac{d\rho}{dz} + \rho(z)\frac{dT}{dz}\right]$$

$$\frac{dT}{dz} = -\frac{g}{R_g} - \frac{T(z)}{\rho(z)} \cdot \frac{d\rho}{dz}$$

But $T(z)$ and $\rho(z)$ are always positive so that when

$$\Gamma \equiv -\frac{dT}{dz} \geq \frac{g}{R_g}$$

then $d\rho/dz \geq 0$. Thus, the autoconvective criteria is that when the density increases with altitude the atmosphere will be forced to convectively adjust and this condition is met when

$$\Gamma_{\text{auto}} \equiv \frac{g}{R_g}$$

Amagats

From the ideal gas law we can define the number of particles at standard temperature (273.15 K) and standard pressure (1 atm = 1013.25 milli-bars). The ideal gas law can be written many ways, here we will adopt the notation(s)

$$P = NkT = \frac{\rho N_a kT}{\overline{\text{mw}}} = \rho R_g T$$

where, N is the number of molecules per unit volume, Boltzmann's constant $k = 1.3806 \cdot 10^{-16}$ erg/K, the gas constant $R_g = N_a \cdot k/\overline{\text{mw}}$, Avogadro's number, $N_a = 6.02 \cdot 10^{23}$ particles per mole, $\overline{\text{mw}}$ is the molecular weight in grams/mole,

This number of molecules at STP is called Loschmidt's number and has the value,

$$N_0 = \frac{P_{stp}}{k \cdot T_{stp}} = 2.687 \cdot 10^{19} \text{cm}^{-3}$$

The number of amagats of a gas is given as a ratio of the actual number of particles at the given temperature and pressure to Loschmidt's number. In planetary atmospheres the number density is a function of height. We can calculate the thickness of an equivalent atmospheric column at standard temperature and pressure. This is denoted as cm-amagats or Km-amagats and is given by

$$\text{Z} = \frac{1}{N_0} \int_z^{\infty} N_i(z) \cdot dz$$

Hydrostatic equilibrium relates the pressure and vertical coordinates

$$dP \equiv -\rho g dz, \quad \text{or } dz = \frac{-dP}{\rho g},$$

and the number density, N_i can be written as a function of the density,

$$N_i = q_i \cdot \frac{\rho N_a}{\overline{\text{mw}}}, \quad \text{molecules/cm}^3$$

where, q_i is the volumetric fraction of species i.

$$\text{Z} = \frac{N_a}{gN_0} \int_0^P \frac{q_i}{\overline{\text{mw}}} \cdot dP \quad \approx \quad \frac{N_a}{gN_0} \frac{q_i}{\overline{\text{mw}}} \cdot P \quad = \quad H_0 \cdot P$$

$$H_0 = \frac{10 \cdot N_a q_i}{gN_0 \overline{\text{mw}}} \frac{\text{Km} - \text{amagat}}{\text{Bar}}$$

	Earth	mars	jupiter	saturn	titan	uranus	neptune	
	-----	-----	-----	-----	-----	-----	-----	
g	981.0	374.1	2425.3	1000.0	136.0	880.1	1110.5	cm/s
q(H2)	1.00	1.00	0.90	0.96	1.00	0.85	0.85	
<mw>	28.97	44.01	2.22	2.14	28.00	2.30	2.30	gm/mole
	-----	-----	-----	-----	-----	-----	-----	
H0	7.88	13.61	37.45	100.50	58.85	94.08	74.56	Km-am/Bar
1000/H0	126.9	73.49	26.70	9.95	16.99	10.63	13.41	mB/Km-am

Beer's Law

$$\tau_\nu^u = e^{-\sec(\theta)\int_z^\infty \kappa_\nu \rho dz}$$

$$\kappa_\nu = \sum \kappa_\nu(CO_2, CO, CH_4, N_2O) \quad + \quad \kappa_\nu(H_2O) \quad + \quad \kappa_\nu(O_3)$$

Transmittance between the top of the atmosphere and level at z is a function of the *quantity* and *absorption* of all gases.

The taller the glass, the darker the brew,
The less the amount of light that comes through.

Beta Plane Approximation

$$f = 2 \cdot \Omega \cdot \sin(\varphi) \approx f_0 + f' \cdot \varphi$$

$$\beta = f' = \frac{2 \cdot \Omega \cdot \cos(\varphi)}{R(\varphi)} = \beta_0 \cdot \frac{R_{eq}}{R(\varphi)} \cdot \cos(\varphi), \quad \beta_0 \equiv \frac{2\Omega}{R_{eq}}$$

	Omega	f(45)	f(60)	f/f_earth	beta0
	rad/s	s-1	s-1		(cm*s)-1
Venus	-2.9921e-07	-4.2314e-07	-4.2314e-07	-0.004103	-9.889e-16
Earth	7.2925e-05	1.0313e-04	1.0313e-04	1.000000	2.287e-13
Mars	7.0852e-05	1.0020e-04	1.0020e-04	0.971583	4.176e-13
Jupiter	1.7585e-04	2.4869e-04	2.4869e-04	2.411438	4.920e-14
Saturn	1.6378e-04	2.3163e-04	2.3163e-04	2.245949	5.435e-14
Uranus	-1.0128e-04	-1.4323e-04	-1.4323e-04	-1.388781	-7.925e-14
Neptune	1.0834e-04	1.5321e-04	1.5321e-04	1.485620	8.730e-14
Titan	4.5608e-06	6.4500e-06	6.4500e-06	0.062541	3.542e-14

Brunt Vaisala or Buoyancy Frequency

$$N^2 = \frac{R_g}{H(z)} \cdot \left[\frac{dT_0}{dz} + \frac{R_g}{C_p}\frac{T_0(z)}{H(z)}\right] = \frac{R_g}{H(z)} \cdot \left[\frac{dT_0}{dz} + \frac{g}{C_p}\right] = \frac{R_g}{H(z)} \cdot \sigma$$

where, $T_0(z) = \overline{T(z, \varphi, \lambda, t)}$, *i.e.*, temperature averaged over longitude λ, latitude φ, and time t. The static stability, σ, is related to the bouyancy frequency as

$$\sigma \equiv \frac{dT_0}{dz} + \frac{g}{C_p}$$

The static stability is positive for stable conditions and zero or negative for unstable conditions.

Cloud clearing, η method

Given the clear radiance, R_{clr} and the fraction of clouds, α_i in scene i.

$$R_i = (1 - \alpha_i) \cdot R_{clr} + \alpha_i \cdot R_{cld}$$

$$R_{clr} = R_1 + \eta \cdot (R_1 - R_2)$$

$$\eta \equiv \frac{\alpha_1}{\alpha_2 - \alpha_1}$$

Chahine, M.T. 1974, J. Atmos. Sci. v.31 p.233

Susskind, J., J. Joiner and C.D. Barnet 1995. Determination of atmospheric and surface parameters from simulated AIRS/AMSU sounding data. I. Retrieval methodology. submitted to J. Atmos. Oceanic Tech. (JAOT) Sep. 1995.

Cloud clearing, N* method

$$N^* \equiv \frac{\alpha_1}{\alpha_2} = \frac{R_1 - R_{clr}}{R_2 - R_{clr}}$$

$$R_{clr} = \frac{R_1 - N^* R_2}{1 - N^*}$$

$$\eta = \frac{N^*}{1 - N^*}$$

McMillin, L.M. and C. Dean 1982. Evaluation of a new operational technique for producing clear radiances. J. Appl. Meteor. v.21 p.1005-1014.

Smith, W.L. 1968. An improved method for calculating tropospheric temperature and moisture from satellite radiometer measurements. Monthly Weather Review v.96 p.387-396.

Clouds, Properties

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from NOAA chart

low clouds:
  up to 6500 ft
  to to 2 Km
  up to 800 mBar (275 K)
  Sc,St,Cu,Cb (Stratocumulus,Stratus,Cumulus,Cumulonimbus)

middle clouds:
  6500 ft to 23,000 ft
  2 Km to 7 Km,  800 mBar to 400 mBar (240 K)
  Ac,As,Ns (Altocumulus, Altostratus, Nimbostratus)

high clouds
  16,500 to 45,000 ft
  5 Km up to 13.75 Km
  530 mBar (255 K) to 140 mBar (216 K)
  Ci,Cc,Cs (Cirrus, Cirrocumulus, Cirrostratus)

                         Reff   Veff  (Hansen, 1971)
  Fair Weather cumulus    5.56  0.111
  Altostratus             7.01  0.113
  Stratus                11.19  0.193
  Cumulus congestus      10.48  0.147
  Stratocumulus           5.33  0.118
  Nimbostratus           10.81  0.143

Earth is typically covered about 50%
average precipitable water is about 2.5 cm
average time aloft is 9 days (from 100 cm of rain/year)
in thunderstorm approximately 10% of water is released

suspended droplets  1 um to 100 um
cloud over continents have smaller droplets
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Dobson Units

$$\text{DOBSON} \equiv 10^{-3}\text{cm} - \text{amagat}$$

given the number density, N, in molecules per cm^3 then

$$\rho_x = \frac{\overline{\text{mw}}_x \cdot N_x(z)}{N_a} \quad \text{grams/cm}^2$$

$$C(L) \equiv \int_z^{z+\Delta z} N(z)dz \simeq \overline{N(L)} \cdot \Delta z = \frac{\overline{N(i)} \cdot \Delta P}{\rho_t \; g} = \frac{q \cdot N_a \cdot \Delta P}{\overline{\text{mw}} \; g} \quad \text{molecules/cm}^2$$

Note that $\rho = q \cdot \rho_t = N \cdot \overline{\text{mw}}/N_a$ where q is the volumetric mixing ratio of the species being measured. To convert to mass column density, M, in grams per cm^2

$$M(L) = \frac{\overline{\text{mw}}_{O_3}}{N_a} \cdot C(L) = \frac{\Delta P}{g}$$

$$\text{DOBSON} = 1000 * Z = \frac{10^3 * \sum C(L)}{N_0}$$

where N_0 is Loschmidt's number.

The atmosphere consists of fixed gases (*e.g.*, CO_2, N_2O, CO), water, and ozone so the pressure within any level is given as

$$\Delta p(L) \equiv \int_{z(L+1)}^{z(L)} \rho_t \; g \; dz = g \int_{z(L+1)}^{z(L)} (\rho_f + \rho_w + \rho_o) dz$$

so that

$$\frac{\Delta p(L) \cdot N_a}{g} = \overline{\text{mw}}_f \cdot C_f(L) + \overline{\text{mw}}_w \cdot C_w(L) + \overline{\text{mw}}_o \cdot C_o(L)$$

$$C_f(L) = \frac{\Delta p(L) \cdot N_a}{\overline{\text{mw}}_f \cdot g} - \frac{\overline{\text{mw}}_w}{\overline{\text{mw}}_f} \cdot C_w(L) - \frac{\overline{\text{mw}}_o}{\overline{\text{mw}}_f} \cdot C_o(L)$$

we can also define the total column density as

$$C_t(L) \equiv \frac{\Delta p(L) \cdot N_a}{\overline{\overline{\text{mw}}}(L) \cdot g}$$

and if we require

$$C_t(L) = C_f(L) + C_w(L) + C_o(L)$$

then,

$$\overline{\overline{\text{mw}}}(L) = \frac{\overline{\text{mw}}_f \cdot C_f(L) + \overline{\text{mw}}_w \cdot C_w(L) + \overline{\text{mw}}_o \cdot C_o(L)}{C_t(L)}$$

$C_t(L)$ in eqn 6 and $\overline{\overline{\text{mw}}}$ can be solved for iteratively with an initial guess of $\overline{\overline{\text{mw}}} = \overline{\text{mw}}_f$

The volumetric mixing ratio, *i.e.*, molecules of species x to the total number of molecules, is given by

$$f_o(L) = \frac{C_o(L)}{C_t(L)} = \frac{C_o(L)}{\Delta P(L) \cdot N_a} \cdot \overline{\overline{\text{mw}}}(L) \cdot g$$

Double Linear Interpolation

$\Delta x = s - s_0, \quad 0 \leq \Delta x \leq 1$
$\Delta y = l - l_0, \quad 0 \leq \Delta y \leq 1$
$s_1 = s_0 + 1, \quad l_1 = l_0 + 1$
$DN(s, l_0) = DN(s_0, l_0) + \Delta x \cdot (DN(s_1, l_0) - DN(s_0, l_0))$
$DN(s, l_1) = DN(s_0, l_1) + \Delta x \cdot (DN(s_1, l_1) - DN(s_0, l_1))$
$DN(s, l) = DN(s, l_0) + \Delta y \cdot (DN(s, l_1) - DN(s, l_0))$
or

$$
\begin{aligned}
DN(s, l) = {} & DN(s_0, l_0) \\
& + \Delta x \cdot (DN(s_1, l_0) - DN(s_0, l_0)) \\
& + \Delta y \cdot (DN(s_0, l_1) - DN(s_0, l_0)) \\
& + \Delta x \Delta y \cdot (DN(s_1, l_1) + DN(s_0, l_0) - DN(s_0, l_1) - DN(s_1, l_0))
\end{aligned}
$$

Earth : molecular abundances

From US Standard Atmosphere 1976, Table 3 and page 33 (Trace Constituents)

Gas	$\overline{mw}$ gm/mole
H	1.00794
C	12.011
N	14.00674
O	15.9994
S	32.066

Gas	$\overline{mw}$ gm/mole	volumetric f_i
N_2	28.0134	0.78084
O_2	31.9988	0.209476
Ar	39.948	0.00934
CO_2	44.00995	314 ppmv
Ne	20.183	18.18 ppmv
He	4.0026	5.24 ppmv
Kr	83.80	1.14 ppmv
Xe	131.30	0.087 ppmv
CH_4	16.04303	1.5 ppmv
H_2	2.01594	0.5 ppmv
$\overline{\overline{mw}}$	28.9644	
H_2O	18.016	$f \approx 0.005$ at 1 km (900 mB)
O_3	47.9982	$f \approx 20$ ppmv at 36 km (5 mB)
N_2O	44.0129	270 ppbv
NO	30.0061	0.5 ppbv
NO_2	46.0055	1 ppbv
H_2S	34.0819	0.05 ppbv
NH_3	17.0306	4 ppbv
SO_2	64.0648	1 ppbv
CO	28.0104	190 ppbv

methane, CH_4
1992 1.7086 ppm
1984 1.6180 ppm $\rightarrow$ +0.0113 ppm/year = +0.7% /year
0.02 ppm peak-to-peak seasonal variation = $\pm$0.01 ppm = $\pm$0.62%
$q_{CH_4} = 1.618 + 0.0113 \cdot (t - 1984.0) \pm 0.01$ ppm

Carbon Dioxide, CO_2
1995 361.0 ppm
1980 336.8 ppm $\rightarrow$ +1.6 ppm/year = +0.475% /year
0.105 ppm peak-to-peak seasonal variation = $\pm$0.052 ppm = $\pm$0.015%
$q_{CO2} = 337 + 1.6 \cdot (t - 1980.0) \pm 0.052$ ppm

$C^{12}O^{16}O^{18} = 4.08 \cdot 10^{-3} \cdot C^{12}O^{16}O^{16}$
$C^{12}O^{16}O^{17} = 7.42 \cdot 10^{-4} \cdot C^{12}O^{16}O^{16}$ Kiehl + Ramanathan 1983. JGR v.88 p.5191.
$C^{13}O^{16}O^{16} = 1.12 \cdot 10^{-2} \cdot C^{12}O^{16}O^{16}$

Earth : molecular spectral regions

CO_2 15 μm spectral region

ν_2 (667.381 bending mode) vibrational-rotational band
ν_1 (1388.23 symmetric stretch mode) fermi-resonance stimulated by ν_2 (618.029, 720.805)
$B_{CO_2} = 0.39$ cm^{-1}
Q-branch at 667.5 (2 mb)
line wings have 1.5 scale height wide weighting functions
line centers have 2.5 scale height weighting functions

CO_2 band correlation parameters at 180 K, Cess + Ramanathan, 1972

band	S at 180 K cm^{-2} amagat^{-1}	A_0 cm^{-1}	γ_0 atm^{-1}cm^{-1}	d cm^{-1}
667	549	17.3	$0.097(300/T)^{2/3}$	1.56
2350	4505	15.4	$0.097(300/T)^{2/3}$	??
3715	102	32.2	$0.097(300/T)^{2/3}$	??

CO_2 band correlation parameters, Kiehl + Ramanathan, 1983

band cm^{-1}	transition $\nu_1\nu_2\nu_3$	S at 300 K cm^{-2}atm^{-1}	E_j cm^{-1}	d cm^{-1}
667.381	$00^00 \rightarrow 01^10$	194	0.0	1.56
667.751	$01^10 \rightarrow 02^20$	15	667.381	0.78
720.805	(I) $01^10 \rightarrow 10^00$	5.00	667.381	1.56
618.029	(II) $01^10 \rightarrow 10^00$	4.27	667.381	1.56
668.107	$02^20 \rightarrow 03^30$	0.85	1335.131	0.78
647.063	(II) $10^00 \rightarrow 11^10$	0.7	1285.410	1.56
668.670	(I) $10^00 \rightarrow 11^10$	0.3	1388.185	1.56

NOTE: for $C^{12}O^{16}O^{17}$ and $C^{12}O^{16}O^{18}$ the mean line spacing is 0.78 for all bands

$$\gamma_0 = 0.067 \left(\frac{300}{T}\right)^{2/3} \quad \text{atm}^{-1}\text{cm}^{-1}$$

$$A_0 = 22.18 \left(\frac{T}{296}\right)^{1/2} \quad \text{cm}^{-1}$$

$$S(T) \simeq S(T_0) \cdot \left(\frac{T_0}{T}\right) \frac{(1 - \exp(-1.439\nu/T))^3}{(1 - \exp(-1.439\nu/T_0))^3}$$

CO_2 4.3 μm spectral region

ν_3 (2349.16 asymmetric stretch mode) band, no Q-branch
high T dependence of high J lines makes narrower weighting functions free of isotopes and hot lines

H_2O 6.3 μm spectral region

band	H_2O	HDO
ν_1	3657.05	2723.68
ν_2	1594.75	1403.49
ν_3	3755.93	3707.47

non-linear molecule with angle of 104.45° $\nu_1 \approx \nu_3 \approx 2 \cdot \nu_2$

O_3 9.6 μm spectral region

ν_1 1110 cm$^-$1

ν_2 1045 cm$^-$1

ν_3 701 cm$^-$1

CH_4 spectrum

of the 9 modes, only ν_1 through ν_4 are independent, and ν_3 and ν_4 are triply degenerate.

^{13}C is 1.108% of all Carbon

The Strongest Bands of Methane (rel. to ground state) f/ Andrews et al.

band	$^{12}CH_4$	$^{13}CH_4$
ν_4	1310.76	1302.77
ν_2	1533.37	
ν_3	3018.92	3009.53
$2\nu_4$	2612	
$\nu_2+\nu_4$	2830	2822
$2\nu_2+\nu_4$	3062	
$\nu_1+\nu_4$	4223	
$\nu_3+\nu_4$	4340	
$\nu_2+\nu_3$	4540	

CH_4 band correlation parameters, Cess + Chen, 1975

band	S cm^{-2} amagat^{-1}	A$_0$ cm^{-1}	γ_0 atm^{-1}cm^{-1}	d cm^{-1}
1306	185	$52(T/300)^{1/2}$	$0.075(300/T)^{1/2}$	5.3
3020	320	$124(T/300)^{1/2}$	$0.075(300/T)^{1/2}$	10.5
4220	20	$124(T/300)^{1/2}$	$0.075(300/T)^{1/2}$	10.5
5861	3	$124(T/300)^{1/2}$	$0.075(300/T)^{1/2}$	10.5

Earth : Oceans

```
Thermocline: ocean layer in which temperature decreases rapidly
             with depth.  From 200 to 1000 meters the temperature
             decreases by 15 degrees F.  In summer the thermocline
             becomes thin and gradients of 1 degree per foot can
             be observed.

 Oceans:   P = g*rho*Z

 P(Bar) = 981cm/s^2 * 12*2.54 * Z(ft) * rho(gm/cm^3) * 1.0E-6 Bar/(dyn/cm^2)
 P(Bar) = 31.0 * Z(1000 ft)

  Z(ft)    rho (gm/cm^3)
     0   1.028  (assume 35 parts-per-thousand salinity)
  1000   1.033
  2000   1.0375
 20000   1.071

      Z    P
     320   10 Bar
   3,200  100 Bar
   3,790  120 Bar   <Z> of all Oceans - depth of life Zone
   4,690  145 Bar  depth of Mediterranian
   4,900  150 Bar   maximum depth of life
   7,282  225 Bar  depth of Gulf of Mexico
  20,000  600 Bar  max. depth of atlantic
  35,600 1100 Bar  1960 depth of Trieste into Chall.II (7 miles!!)

Note: ratio of Volume of Ocean to Land (above Z=0) = 11
      ratio of Area of Ocean to Land:
        N   61:39 = 3:2
        S   81:19 = 4:1
      all  142:58 = 5:2

source: 1982 Encyclopedia Brittanica
```

Equations of motion

Momentum

$$\frac{d\vec{V}}{dt} = -2\Omega \times \vec{V} - \frac{1}{\rho}\nabla P + \hat{k}g + \vec{F}$$

$$\vec{V} \equiv \hat{\imath}u + \hat{\jmath}v + \hat{k}w$$

$$\frac{du}{dt} - 2\Omega v \sin(\varphi) = \frac{-1}{\rho}\frac{\partial P}{\partial x} + \frac{uv\tan(\varphi)}{a} - \frac{uw}{a} - 2\Omega w \cos(\varphi) + F_x$$

$$\frac{dv}{dt} + 2\Omega u \sin(\varphi) = \frac{-1}{\rho}\frac{\partial P}{\partial y} - \frac{u^2\tan(\varphi)}{a} - \frac{vw}{a} + F_y$$

$$\frac{dw}{dt} - 2\Omega u \cos(\varphi) = \frac{-1}{\rho}\frac{\partial P}{\partial z} + \frac{u^2 + v^2}{a} - g + F_z$$

Continuity

$$\frac{1}{\rho}\frac{d\rho}{dt} + \vec{\nabla} \cdot \vec{V} = 0$$

when motions are on the order of a scale height or less

$$\vec{\nabla} \cdot (\rho_0 \vec{V}) = 0$$

Energy

$$\frac{\dot{q}}{T} = c_p \frac{d\log_e(\Theta)}{dt} = \frac{c_p}{T}\frac{dT}{dt} - \frac{R_{univ}}{P}\frac{dP}{dt}$$

FFT

Forward FFT

$$F(u) = \mathbf{FFT}(f(x), -1) \equiv \frac{1}{m} \sum_{x=0}^{m-1} f(x) \cdot e^{-i \cdot u \cdot \frac{2\pi x}{m}} \quad \text{for } u = 0, m-1$$

Inverse FFT

$$f(x) = \mathbf{FFT}(F(u), +1) \equiv \sum_{u=0}^{m-1} F(u) \cdot e^{i \cdot x \cdot \frac{2\pi u}{m}} \quad \text{for } x = 0, m-1$$

Forward Cosine transform

$$F(u) \equiv \frac{1}{m} \cdot \int_0^m f(x) \cdot \cos(u \cdot \frac{\pi x}{m}) \cdot dx \approx \frac{f(0) + f(m) \cdot \cos(\pi u)}{2m} + \frac{1}{m} \sum_{x=1}^{m-1} f(x) \cdot \cos(u \cdot \frac{\pi x}{m}) = \mathbf{FFT}(g(x), -1)$$

$g(x) = f(x) \quad \text{for } x = 0, m$
$g(2m - x) = f(x) \quad \text{for } x = 1, m-1$

Inverse Cosine Transform

$$f(x) = F(0) + F(m) \cdot \cos(\pi x) + 2 \cdot \sum_{u=1}^{m-1} \left(F(u) \cdot \cos(x \cdot \frac{\pi u}{m}) \right) = \mathbf{FFT}(G(u), +1)$$

$G(x) = F(x) \quad \text{for } x = 0, m$
$G(2m - x) = F(x) \quad \text{for } x = 1, m-1$

Gas Constant

$$R_{univ} = c_p - c_v = 8.3143 \text{ Joules/mole/K} = 8.3143 \cdot 10^7 \text{ erg/mole/K}$$

$$\overline{\text{mw}}_{air} = 28.964 \text{ gm/mole}$$

$$R_g(air) = R_{univ}/\overline{\text{mw}}_{air} = 287.05 \text{ Joule/Kg/K}$$

$$C_p(air) = 1005 \text{ J/Kg/K}$$

$$\kappa \equiv \frac{R_g}{C_p} = \frac{R_{univ}}{c_p}$$

$$\gamma \equiv \frac{C_p}{C_v} = \frac{c_p}{c_v}$$

	<mw>	R_gas	C_p	kappa	gamma
	gm/mole	J/gm/K	J/gm/K		
Venus	44.01	0.18892	0.8501	0.2222	1.2857
Earth	28.96	0.28710	1.0040	0.2860	1.4005
Mars	44.01	0.18892	0.8312	0.2273	1.2941
Jupiter	2.22	3.74518	12.3591	0.3030	1.4348
Saturn	2.14	3.89246	14.0129	0.2778	1.3846
Uranus	2.30	3.61491	13.0137	0.2778	1.3846
Neptune	2.30	3.61491	13.0137	0.2778	1.3846
Titan	28.67	0.29000	1.0440	0.2778	1.3846

Geopotential

$$d\Phi = -g \cdot dz$$

$$\Phi = \int_0^z d\Phi$$

Geostrophic Approximation

Horizontal motions are in balance with the pressure gradient force. Friction is negligible (i.e., far away from surface). Steady flow with small curvature (i.e., $dV/dt = 0$). Works well for $z \geq 1$ km, $\varphi > 10°$.

$$-\rho \cdot f \cdot V_g = \frac{\partial P}{\partial x}$$

$$\rho \cdot f \cdot U_g = \frac{\partial P}{\partial y}$$

Gravity

Gravitational Forces

	<g>	<g>	g(0)	g(45)	g(90)
	cm/s**2	g_earth	cm/s**2	cm/s**2	cm/s**2
Venus	889.89	0.9082	889.89	889.89	889.89
Earth	979.86	1.0000	976.81	981.29	987.16
Mars	374.10	0.3818	372.40	374.90	378.10
Jupiter	2425.61	2.4755	2256.64	2504.58	2833.44
Saturn	1000.09	1.0206	882.36	1055.28	1283.11
Uranus	880.07	0.8982	860.29	889.23	928.60
Neptune	1110.46	1.1333	1087.16	1121.31	1167.07
Titan	135.80	0.1386	135.80	135.80	135.80

$$q = \frac{\omega^2 \cdot a^3}{G \cdot M}$$

	10^6*J2	10^6*J4	10^6*J6	10^6*q
Venus	.	.	.	.
Earth	.	.	.	.
Mars	1960.454	.	.	.
Jupiter	14697	-584.0	31.0	89180
Saturn	16331	-914.0	108.0	154766
Uranus	3516	-31.9	0.0	29513
Neptune	4000	0.0	0.0	28960
Titan	.	.	.	.

Hubbard and Marley (1989)
Kieffer

Heat Balance

$$F_{sun} = \frac{S_0}{r_s^2} \quad S_0 \equiv 1.37 \cdot 10^6 \text{ ergs/cm}^2$$

$$P_{abs} = (1 - A_b) \cdot F_{sun} \cdot \text{sunlit area} = (1 - A_b) \cdot F_{sun} \cdot \pi \cdot R_{eq} \cdot R_{pl}$$

$$a = \text{total area} = 2\pi \cdot R_{eq}^2 + \pi \frac{R_{pl}^2}{e} \cdot \log_e \left(\frac{1+e}{1-e} \right)$$

$$e = \frac{\sqrt{R_{eq}^2 - R_{pl}^2}}{R_{eq}}$$

$$F_{abs} = \frac{P_{abs}}{a}$$

$$T_{eq} = \left(\frac{P_{abs}}{a \cdot \sigma} \right)^{\frac{1}{4}}$$

If the eccentricity is equal to zero, $e = 0$, then

$$a = 4\pi \cdot R_{eq} \cdot R_{eq}$$

$$F_{abs} = (1 - A_b) \cdot \frac{F_{sun}}{4}$$

$$T_{eq} = \left(\frac{(1 - A_b) \cdot F_{sun}}{4 \cdot \sigma} \right)^{\frac{1}{4}}$$

	bond albedo	F_sun erg/s/cm^2	F_abs erg/s/cm^2	P_abs erg/s	Teq K
Venus	0.750	2618462.8	163653.9	7.5299e+23	231.89
Earth	0.306	1370000.0	237414.7	1.2110e+24	254.49
Mars	0.250	590102.3	110447.6	1.5922e+23	210.18
Jupiter	0.343	50616.0	8123.3	4.9933e+24	109.45
Saturn	0.342	15011.7	2381.4	1.0167e+24	80.54
Uranus	0.300	3691.7	641.0	5.1818e+22	58.01
Neptune	0.290	1498.8	264.0	2.0141e+22	46.47
Titan	0.220	15011.7	2927.3	2.4391e+21	84.80

$$P_{int} = F_{int} \cdot a$$

$$F_{out} = F_{int} + F_{abs}$$

$$\text{ratio} = \frac{F_{out}}{F_{abs}} = \frac{F_{int} + F_{abs}}{F_{abs}}$$

$$T_{eff} = \left(\frac{F_{out}}{\sigma} \right)^{\frac{1}{4}}$$

	F_int	P_int	F_abs+F_int	Teff	out/in
	erg/s/cm^2	erg/s	erg/s/cm^2	K	ratio
Venus	0.0	0.0000e+00	163653.9	231.89	1.0000
Earth	62.0	3.1624e+20	237476.7	254.51	1.0003
Mars	0.0	0.0000e+00	110447.6	210.18	1.0000
Jupiter	5440.0	3.3439e+24	13563.3	124.42	1.6697
Saturn	2010.0	8.5815e+23	4391.4	93.85	1.8440
Uranus	42.0	3.3953e+21	683.0	58.94	1.0655
Neptune	433.0	3.3029e+22	697.0	59.24	2.6399
Titan	0.0	0.0000e+00	2927.3	84.80	1.0000

Humidity

Absolute humidity

From the ideal gas law, the density of water, ρ_w is given by

$$\rho_w = \frac{\overline{\mathrm{mw}}_w f_w e}{R_* T}$$

where $\overline{\mathrm{mw}}_w$ is the molecular weight of water (18.016 gm/mole), f_w is a correction factor for non-ideal behaviour which can be taken as $f_w = 1$, e is the partial pressure of water, R_* is the universal gas constant, and T is the local temperature.

Specific humidity

The total density, ρ, is given by the sum of dry and moist densities (*i.e.*, assumes all other species are included in "dry").

$$\rho = \rho_d + \rho_w = \frac{\overline{\mathrm{mw}}_d \cdot (P - f_w e)}{R_* T} + \frac{\overline{\mathrm{mw}}_w f_w e}{R_* T} = \frac{\overline{\mathrm{mw}}_d \cdot (P - 0.37803 f_w e)}{R_* T}$$

where $\overline{\mathrm{mw}}_d$ is the molecular weight of dry air (28.966), $\frac{\overline{\mathrm{mw}}_w}{\overline{\mathrm{mw}}_d} = 0.62197$, and $(\overline{\mathrm{mw}}_d - \overline{\mathrm{mw}}_w)/\overline{\mathrm{mw}}_d = 0.37803$. The specific humidity is the ratio of the moist to total density and is dimensionless, but is usually expressed in gm/Kg units.

$$q = \frac{\rho_w}{\rho} = \frac{0.62197 f_w e}{P - 0.37803 f_w e} \simeq 0.622 \frac{e}{P}$$

Note that

$$\rho_w = q \cdot \rho$$

and the volumetric mixing ratio, f, (ı.t., ratio of water number density, N_w, to the total number density N_t) is given by

$$f_w \equiv \frac{N_w}{N_t} = \frac{\overline{\mathrm{mw}}}{\overline{\mathrm{mw}}_w} \cdot q$$

where the average molecular weight, $\overline{\mathrm{mw}}$, is given by

$$\overline{\mathrm{mw}} = \overline{\mathrm{mw}}_d \cdot (1 - f_w) + \overline{\mathrm{mw}}_w \cdot f_w$$

Mass Mixing Ratio

$$r = \frac{\rho_w}{\rho_d} = 0.62197 \cdot \frac{f_w e}{(P - f_w e)} \simeq 0.62197 \cdot \frac{e}{(P - e)} \approx q$$

from US Standard Atmosphere 1976, Table 20, pg. 44, $r(z)$ ppm by mass

Alt (km)	P (mB)	record low	1% low	midlat. mean	1% high	record high
sfc	1013.25	0.1	5.0	4,686	30,000	35,000
1	890	24.0	27.0	3,700	29,000	31,000
2	790	21.0	31.0	2,843	24,000	28,000
4	610	16.0	24.0	1,268	18,000	22,000
6	470	6.2	12.0	554	7,700	8,900
8	350	6.1	6.1	216	4,300	4,700
10	265		5.3	43.2	1,300	
12	195		1.2	11.3	230	
14	140		1.5	3.3	48	
16	100		1.0	3.3	38	

%Relative Humidity

$$U \equiv 100 \cdot \frac{r}{r_s} \simeq 100\frac{q}{q_s} \simeq 100\frac{e}{e_s}$$

where $r_s \equiv r(e = e_s)$ and $q_s \equiv q(e = e_s)$. The saturated vapor pressure is approximately given by

$$e_s \simeq 6.11 \cdot 10^{\frac{aT}{b+T}} \quad \text{mBar}$$

$$r_s \simeq 0.62197 \cdot \frac{e_s}{P - e_s} \quad \text{for} \quad e_s > P$$

$$r_s \simeq 0.62197 \quad \text{for} \quad e_s \geq P$$

where T is given in °C and a and b are constants.

	a	b
above water	7.5	237.3
above ice	9.5	265.5

Dew Point Temperature

The Dew point is the temperature at which the partial pressure of water reaches the saturation value, that is

$$e = \frac{\rho_w R_* T}{\overline{\text{mw}}_w f_w} = e_s(T_{dp})$$

The empirical expression for e_s can be used or a table lookup can be utilized after e is calculated.

Hurricanes

Tropical Classification

Gale force winds	(> 15 m/s)
Tropical Depression	(20-34kts and a closed circulation)
Tropical Storm (named)	(35-64kts)
Hurricane	(65+kts or 74+mph)

Saffir-Simpson Scale

	knots	m/h	Ps,(mB)	inch.Hg
Category 1	64- 83	74- 95	>980	>28.94
Category 2	83- 95	96-110	965-979	28.50-28.91
Category 3	96-113	111-130	945-964	27.91-28.47
Category 4	114-135	131-155	920-944	27.17-27.88
Category 5	>135	>155	<920	<27.16

Note: 1 knot = 1 nautical mile/hour = 1.151555 mile/hour = 1.853248 Km/hr

Storm	Date	Cat	Pc	Rc	winds
Nancy	Sep.12,1961		888		185 kt
Tip	Oct.12,1979		870		85 m/s
Camille	1969	5	909		165 kt
Allen	1980				165 kt
Gilbert	Sep. 1988		888		
Hugo	1989	4			
Andrew	1992	4	922		

Empirical formula for approximate energy, ergs, of a storm with a central pressure, P_c, given in mBar and a radius of nearest circular isobar, R_c, given in kilometers is

$$E = 0.712 \cdot 10^{22} \cdot (1010 - P_c) \cdot \left(\frac{R_c}{111}\right)^2 \quad \text{ergs}$$

Hydrostatic Equilibrium

$$\partial P \equiv -g \cdot \rho \cdot dz = -g \cdot \frac{n \cdot \overline{\mathrm{mw}}}{V} \cdot dz$$

If the Ideal gas law holds (note: $R_g = R_{univ}/\overline{\mathrm{mw}}$), then

$$\partial P = -P \cdot \frac{g \cdot \overline{\mathrm{mw}}}{R_{univ} T} \cdot dz = -P \cdot \frac{dz}{H(z)}, \quad \text{where} \quad H(z) = \frac{R_g(z) \cdot T(z)}{g(z)}$$

If the scale height is constant, then

$$P = P_0 \cdot \exp(-z/H_0), \quad \text{where} \quad H_0 = \frac{R_g \cdot T}{g}$$

Ideal Gas Law

$$P \cdot V = N \cdot k \cdot T = n \cdot R \cdot T$$

where N is the total number of molecules, n is the total number of moles, $N_a = 6.02217 \cdot 10^{23}$molecules/mole, $R_{univ} = 8.314 \cdot 10^7$ erg/K/mole, $k = R_{univ}/N_a = 1.380622 \cdot 10^{-16}$ erg/K

$$P = \left(\frac{N}{V}\right) kT = \left(\frac{\rho \cdot N_a}{\overline{\mathrm{mw}}}\right) kT$$

$R_g \equiv R_{univ}/\overline{\mathrm{mw}}$ erg/K/gm

$$P = \rho \cdot \frac{R_{univ}}{\overline{\mathrm{mw}}} \cdot T = \rho R_g T$$

IR filter bandpasses

	um	range	trans	thick
J	1.25	1.1 - 1.4	60%	2.0mm
H	1.65	1.5 - 1.8	60%	2.0mm
K	2.20	2.0 - 2.4	60%	1.5mm
L	3.82	3.5 - 4.15	60%	1.0mm
M	4.70	4.4 - 5.0	60%	1.0mm

Latitude : planeto – graphic and planeto – centric

centric latitude, θ and the planeto-graphic latitude, φ

$$\beta \equiv \frac{R_{eq}}{R_{pl}}$$

$$\theta = \tan^{-1}\left(\tan(\varphi)/\beta^2\right)$$

$$\varphi = \tan^{-1}\left(\tan(\theta)\cdot\beta^2\right)$$

$$R(\theta) = \frac{R_{eq}R_{pl}}{\sqrt{R_{eq}^2\cdot\sin^2(\theta)+R_{pl}^2\cdot\cos^2(\theta)}}$$

$$f = \frac{R_{eq}-R_{pl}}{R_{eq}}$$

$$e = \frac{\sqrt{R_{eq}^2-R_{pl}^2}}{R_{eq}} = \sqrt{1-\left(\frac{R_{pl}}{R_{eq}}\right)^2}$$

$$S = \text{total area} = 2\pi\cdot R_{eq}^2 + \pi\frac{R_{pl}^2}{e}\cdot\log_e\left(\frac{1+e}{1-e}\right)$$

$$V = \text{total volume} = \frac{4}{3}\pi R_{eq}^2\cdot R_{pl}^2$$

If $e = 0$ then

$$S = 4\pi\cdot R_{eq}\cdot R_{eq}$$

CRC Standard Mathematical Tables 26^{th} edition, pg. 129. The volume of an oblate sphere is equal to a sphere with radius $\overline{R}$.

$$\overline{R} = \left(R_{eq}^2\cdot R_{pl}\right)^{\frac{1}{3}}$$

	Req	Rpl	Req/Rpl	f	e	<R>
	Km	Km				Km
Venus	6051.0	6051.0	1.000000	0.000000	0.000000	6051.00
Earth	6378.5	6356.0	1.003540	0.003527	0.083920	6370.99
Mars	3393.0	3375.0	1.005333	0.005305	0.102869	3386.99
Jupiter	71492.0	66854.0	1.069375	0.064874	0.354316	69911.33
Saturn	60268.0	54364.0	1.108601	0.097962	0.431658	58232.01
Uranus	25559.0	24973.0	1.023465	0.022927	0.212906	25362.16
Neptune	24820.0	24274.0	1.022493	0.021998	0.208597	24636.66
Titan	2575.0	2575.0	1.000000	0.000000	0.000000	2575.00

Longitude Systems and conversion

P_1 = reference rotational period of system # 1
P_2 = reference rotational period of system # 2
$L_1(t-t_0)$ is a known system # 1 longitude relative to time t_0
$L_2(t-t_0)$ is the desired longitude value in system # 2 relative to time=t_0

$$L_2(t-t_0) = L_1(t-t_0) + L_{21} + (t-t_0) \cdot \left[\frac{24 \cdot 360}{P_2} - \frac{24 \cdot 360}{P_1}\right]$$

L_{21} is a reference longitude. It is defined as:
$L_{21} = L_2(t=t_0) - L_1(t=t_0)$

Jupiter

$S_I = 9^h\ 50^m\ 30.003^s = 877.90°/24^h$. Used for equatorial motions.
$S_{II} = 9^h\ 55^m\ 40.632^s = 870.27°/24^h$. Used for mid-latitudes and GRS.
$S_{III} = 9^h\ 55^m\ 29.711^s \pm 0.04^s = 9.9249197^h \pm 0.000011^h = 870.536°/24^h \pm 0.000966°/24^h$
$\lambda_I = 67.10 + 877.900 \cdot (t-t_0)$
$\lambda_{II} = 43.30 + 870.270 \cdot (t-t_0)$
$\lambda_{III} = 284.95 + 870.536 \cdot (t-t_0)$

$$\lambda(S_{III}) = \lambda(S_I) + 217.85 - 7.364 \cdot (t-t_0)$$

$$\lambda(S_{III}) = \lambda(S_{II}) + 241.65 - 0.266 \cdot (t-t_0)$$

where $t_o = 2451545.0$ TDB or Jan. 1.5, 2000.
$U_{III}(S_I) = 106.35$ m/s
$U_{III}(S_{II}) = -3.4$ m/s

Saturn

$S_{III} = 10^h\ 39^m\ 22.4^s \pm 7^s = 10.656222^h \pm 0.00194^h = 810.7939024°/24^h \pm 0.148°/24^h$
$\lambda_{III} = 38.90 + 810.793902 \cdot (t-t_0)$
$\lambda_I = 227.2037 + 844.3 \cdot (t-t_0)$

$$\lambda(S_{III}) = \lambda(S_I) + 171.6963 - 33.5061 \cdot (t-t_0)$$

$U_{III}(S_I) = 410.36$ m/s
For the Voyager encounters the reference time should be set to the closest approach time. Other possibilies are:

	P1	P2	FDS(t=0)	L21
Neptune	17.866	16.11	11200.0	-334.5365

Orbital : Equations of Motion

$$\overline{r} = \frac{a}{\frac{1+e^2}{2}}$$

Orbits : Earth

$$V = \sqrt{\frac{G \cdot M_e}{r}} \quad \text{cm/sec}$$

where,

$G = 6.67 \cdot 10^{-8}$ dyne cm^2/ gm^2
$M_e = 5.975 \cdot 10^{27}$ grams
$r = 10^5 \cdot (\overline{R_e} + z)$
$R_e = 6374.87$

$$P = \frac{2 \cdot \pi \cdot r}{V \cdot 3600} \quad \text{hours}$$

$$V_g = \frac{2 \cdot \pi \cdot 10^5 \cdot R_e}{P \cdot 3600} \quad \text{cm/sec}$$

r(Km)	P(hr)	V(Km/s)	Vg(Km/s)
200.0	1.474	7.786	7.549
400.0	1.542	7.670	7.217
700.0	1.645	7.505	6.763
900.0	1.715	7.401	6.486
35863.9	24.000	3.072	0.464

Planck function

The Planck function, $B_\nu(T)$, is given by

$$B_\nu(T) = \frac{\alpha_1 \nu^3}{\exp(\frac{\alpha_2 \nu}{T}) - 1}$$

where,

ν = wavenumber in $\mathrm{cm}^{-1} = 10^4/\lambda$, λ in μm.
T = temperature in degrees Kelvin
$\alpha_1 = 2hc^2 = 1.191066 \cdot 10^{-5}$ for radiance in units of mW $\cdot \mathrm{m}^{-2} \cdot \mathrm{steradian}^{-1}/\mathrm{cm}^{-1}$.
$\alpha_2 = hc/k = 1.438833 \quad \mathrm{K}/\mathrm{cm}^{-1}$
h is Planck's constant (6.62620 $\cdot 10^{-34}$ Joule
c is the speed of light (2.99793 $\cdot 10^8$ m/second)
k is Boltzmann's constant (1.38062 $\cdot 10^{-23}$ Joule/K)
σ is the Stefan-Boltzmann constant $= 5.67 \cdot 10^{-5} \quad \mathrm{erg} \cdot \mathrm{s}^{-1} \cdot \mathrm{cm}^{-2} \cdot \mathrm{K}^{-4} \quad or \quad \mathrm{mW} \cdot \mathrm{m}^{-2} \cdot \mathrm{K}^{-4}$

$$\text{Radiant Energy} = A \cdot \sigma \cdot T^4$$

The brightness temperature, T_b, of a given radiance, R_ν, is found with the inverse of the Planck function.

$$T_b \equiv B_\nu^{-1}(R_\nu) = \frac{\alpha_2 \cdot \nu}{\log_e \left(1 + \frac{\alpha_1 \nu^3}{R_\nu}\right)}$$

The derivative of the Planck function is given by

$$\frac{\partial B_\nu}{\partial T} = \frac{\alpha_1 \alpha_2 \nu^4}{T^2} \cdot \frac{\exp(\alpha_2 \nu / T)}{(\exp(\alpha_2 \nu / T) - 1)^2}$$

Infrared approximation: for $\nu \geq 600 \mathrm{cm}^{-1}$ and $T \leq 300$ K.

$$B_\nu(T) \simeq \alpha_1 \nu^3 \exp(-\alpha_2 \nu / T) \qquad \mathrm{mW} \cdot \mathrm{m}^{-2} \cdot \mathrm{steradian}^{-1} \cdot (\mathrm{cm}^{-1})^{-1}$$

$$T_b \simeq \frac{-\alpha_2 \cdot \nu}{\log_e \left(\frac{\alpha_1 \nu^3}{R_\nu}\right)}$$

$$\frac{\partial B_\nu}{\partial T} \approx \frac{\alpha_1 \alpha_2 \nu^4}{T^2} \cdot \exp(-\alpha_2 \nu / T)$$

Microwave Rayleigh Jeans approximation: $\lambda(\mathrm{mm}) = 300/f$, $\nu = f/30\ \mathrm{cm}^-1$, $\quad f$ is in GHz

Wavenumbers, ν, are frequency units and are assumed to be in vacuum while wavelength is the wavelength specified within the medium. Typically, wavelengths are expressed as wavelength in air, λ_a or wavelength in vacuum, λ_v.

$$\nu \equiv \frac{10000}{\lambda_v}$$

$$\lambda_v = n_a \cdot \lambda_a$$

$$\nu = \frac{10000}{n_a \cdot \lambda_a} \quad n_a = 1.00027$$

```
      f        cm-1        um       mm
 0.66 GHz     0.02    454545   454.55   P band (SAR)
 1.25 GHz     0.04    240000   240.00   C band (SAR)
 5.33 GHz     0.18     56285    56.29   L band (SAR)
 6.6  GHz     0.22     45454    45.45   MIMR (surface)
   23 GHz     0.77     13043    13.04   AMSU-A
   50 GHz     1.67      6000     6.00   AMSU-A
   60 GHz     2.00      5000     5.00   AMSU-A
   89 GHz     2.97      3371     3.37   AMSU-C
  118 GHz     3.93      2542     2.54   MHS-X
  183 GHz     6.10      1639     1.64   AMSU-B
```

$$B_\nu \simeq \frac{\alpha_1}{\alpha_2 \cdot c^2 \cdot 10^{-18}} \cdot f^2 \cdot T \quad = \quad 9.2105 \cdot 10^{-9} \cdot f^2 \cdot T \qquad \mathrm{mW} \cdot \mathrm{m}^{-2} \cdot \mathrm{steradian}^{-1} \cdot (\mathrm{cm}^{-1})^{-1}$$

$$T_b \simeq 1.0857 \cdot 10^8 \frac{R_\nu}{f^2}$$

$$\frac{\partial B_\nu}{\partial T} \approx 2 \cdot c \cdot k \cdot \nu^2 = \frac{\alpha_1}{\alpha_2} \cdot \nu^2 = \frac{\alpha_1}{\alpha_2 \cdot c^2 \cdot 10^{-18}} \cdot f^2$$

wavelength mm	frequency GHz	B(T=300K) mW/m^2/ster/cm^{-1}	dB/dT mW/m^2/ster/cm^{-1} / K	(1/B) · dB/dT %/K
6.0	50	0.007	0.000023	0.335
3.0	100	0.027	0.000092	0.336
2.0	150	0.061	0.000207	0.337
1.5	200	0.109	0.000368	0.338

wavelength μ m	wavenumber cm^{-1}	B(T=300K) mW/m^2/ster/cm^{-1}	dB/dT mW/m^2/ster/cm^{-1} / K	(1/B) · dB/dT %/K
16.7	600	153.38	1.559	1.0
9.1	1100	81.49	1.441	1.8
6.2	1600	22.69	0.581	2.6
4.3	2300	2.35	0.086	3.7
3.7	2700	0.56	0.024	4.3
3.3	3000	0.18	0.009	4.8

Planetary Missions

Mars

```
1965 Mariner 4 : 22 low resolution pictures
1969 Mariner 6 : flyby
1969 Mariner 7 : flyby
1971 Mariner 9 : orbiter (resol. = .1-1 km)
1976 Vikings : 55,000 images, resol. 150-300 meters
     Viking 1 orbiter  (4 yrs)
     Viking 1 lander  22.27N 47.97W
                       July 20, 1976 4:13 pm LMT
               T=250 K
     Viking 2 orbiter
     Viking 2 lander  47.57N 225.74W
                       Sept. 3, 1976 9:49 am LMT
               T=150 K diurnal variation 35-50K
Maritian Year :
     N. polar cap   H2O ice
     S. polar cap   CO2 ice
     Olympus Mons   600 km diameter volcano,  26 km tall
                    4 km high scarp (cliff) rings its base
     ``face'' on Mars is located in the Cydonia Mensae region
       at roughly 40.9 degrees North latitude and 9.45 degrees
       West longitude.
```

Outer Planet Mission Summary

Planet	Craft	Date	Julian	Ls
Jupiter	Pioneer 10	12/ 3/73	2442019.5	1.4
	Pioneer 11	12/ 2/74	2442383.5	33.9
	Voyager I	3/ 5/79	2443937.5	170.4
	Voyager II	7/ 9/79	2444063.5	180.4
Saturn	Pioneer 11	9/ 1/79	2444117.5	354.1
	Voyager I	11/12/80	2444555.5	8.9
	Voyager II	8/25/81	2444841.5	18.4
Uranus	Voyager II	1/24/86	2446454.5	271.3
Neptune	Voyager II	8/25/89	2447753.5	243.3

Potential Temperature

Derived directly from integration of the 1^{st} law of thermodynamics. It is the temperature a parcel of air at P and T would have if it were at P_s. It is conserved for adiabatic motions, (*i.e.,* $d\Theta/dt = 0$).

$$\frac{dQ}{n \cdot \overline{\mathrm{mw}} \cdot T} = \frac{C_p dT}{T} - \frac{R_g dP}{P} = 0$$

$$\Theta = T \cdot \left(\frac{P_s}{P}\right)^{\kappa} = \frac{P}{\rho R_g}\left(\frac{P_s}{P}\right)^{\kappa} \qquad \kappa = R_g/C_p$$

For earth $\kappa = 0.286$ ($\overline{\mathrm{mw}} = 28.96, C_p = 1.004$ Joules/gram/K). Some authors write this equation with $\gamma = C_p/C_v = 1/(1-\kappa)$

$$\frac{\partial P}{\partial z} = -\rho \cdot g = \frac{P \cdot g}{R_g \cdot T} = -\frac{P}{H(z)}$$

$$\partial \log(P) = -H(z) \cdot \partial z$$

therefore,

$$\frac{P}{P_s} = e^{-\int_0^z dz'/H(z')}$$

$$\Theta = T \cdot e^{-\int_0^z dz'/H(z')}$$

if $H(z) = H_0$

$$\Theta = T \cdot e^{-\kappa \cdot z/H_0}$$

Potential Vorticity

For adiabatic motion (*i.e.*, Θ is a constant) the density is $\rho = P^{(C_v/C_p)}$. For motions between layers of constant Θ

$$q = (\zeta + f) \cdot \frac{\partial \Theta}{\partial p} = \text{constant}$$

In the Shallow Water Model, where ρ is a constant (*i.e.*, incompressible), the conserved quantity is usually written as:

$$q \equiv \frac{\zeta + f}{gh}, \quad \text{seconds/meter}^2$$

example : flow over a mountain
$< u >$ is eastward (westerly) then flow turns towards equator as it passes mountain, then oscillates around original latitude.
$< u >$ is westward (easterly) then flow turns equatorward (note flow turning poleward 'bucks' the flow causing it to turn equatorward. System is returned to original latitude after it passes mountain. No oscillations.

Radiative time constant

The volumetric heating rate, Q

$$\frac{\partial Q}{\partial T} \equiv \frac{1}{\tau_r}$$

$$Q_{ir} = \sigma \cdot T^4 \cdot \frac{\partial \epsilon}{\partial z}$$

$$\tau_r = \frac{\rho C_p}{4\sigma T^3 \frac{\partial \epsilon}{\partial z}}$$

Harshvardan and Cess (1976, Tellus **28** p.1-10) give a value for Earth CO_2 of $\partial\epsilon/\partial z = -0.0081$ per Km and Cess and Khetan (1973, JQSRT **13** p.995-1009) give a value for Jupiter,Saturn,Neptune, and Uranus of $\partial\epsilon/\partial z = -0.0068, -0.0063, -0.0061, \text{and} -0.0058$ per Km, respectively.

$$\tau_r = \frac{\rho H C_p}{4\epsilon\sigma T^3}$$

The phase shift, Φ, is the *seasonal* lag due to the radiative time constant. It arises from the 1^{st} Fourier moment of the thermal response. A value of $\Phi = 90°$ indicates a full seasonal shift (*e.g.*, the warm maximum expected at the summer solstice would occur at the autumnal equinox).

$$\Phi = \tan^{-1}\left(2\pi \cdot \frac{\tau_r}{\tau_o}\right)$$

where τ_o is the orbital period. For $\epsilon = 0.3$ the values are:

	Teff	Peff	rho	H	tau_rad	phase
	K	mBar	gm/cm^3	Km	years	deg
Venus	229.0	20.000	4.6230e-05	4.862	0.007	3.94
Earth	255.0	500.000	6.8297e-04	7.471	0.131	39.47
Mars	210.0	6.000	1.5124e-05	10.605	0.006	1.17
Jupiter	124.0	400.000	8.6132e-05	19.146	4.534	67.39
Saturn	95.0	300.000	8.1128e-05	36.975	20.794	77.27
Uranus	59.0	430.000	2.0161e-04	24.234	131.311	84.15
Neptune	59.0	500.000	2.3443e-04	19.206	121.008	77.67
Titan	85.0	1.000	4.0568e-06	18.152	0.053	82.54

Radiative transfer Equation

$$\Theta_{i,calc} = B_\nu^{-1}(R_{i,calc})$$

$$R_{i,calc} = \epsilon_i B_i(T_s) + \rho_i' H_i \tau_i'(p_s)\cos(\theta_0) + \int_{p=p_s}^{p=0} B_i(T(p))\left(\frac{d\tau_i^u}{d\ln p}\right) d\ln p + \rho_i \tau_i^u(p_s) \int_{p=p_s}^{p=0} B_i(T(p))\left(\frac{d\tau_i^d}{d\ln p}\right) d\ln p$$

$B_i(T)$ is the Planck function evaluated at the effective channel wavenumber.
H_i is the channel averaged solar irradiance at the top of the atmosphere.
θ_0 is the local zenith angle of the Sun.
τ_i' is the channel averaged two path transmittance from the Sun to the surface to the satellite.
$\tau_i^u(p)$ is the atmospheric transmittance measured between p and the top of the atmosphere for channel i.
$\tau_i^d(p)$ is the atmospheric transmittance between p and the surface for channel i.

Explicit retrieved parameters
ϵ_i is the surface emissivity at ν_i.
T_s is the surface temperature.
$T(p)$ is the surface temperature profile.
ρ_i is the surface spectral reflectivity at ν_i.
ρ_i' is the surface spectral bidirectional reflectance of solar radiation at ν_i.

Implicit retrieved parameters ($i.e,$ within τ_i and τ_i')
$CO_2(p)$ is the carbon dioxide profile.
$q(p)$ is the humidity (water) profile.
$O_3(p)$ is the ozone profile.
$CO(p)$ is the carbon monoxide profile.
$CH_4(p)$ is the methane profile.
$N_2O(p)$ is the nitrogen dioxide profile.

Rayleigh Scattering : Optical Depth

The average scattering cross section per particle is given by

$$\sigma(\lambda) = \frac{128\pi^5\alpha^2}{3\lambda^4} \cdot \frac{6+3\delta}{6-7\delta}$$

$$\alpha = \frac{n-1}{2\pi N_0} \approx \frac{n^2-1}{4\pi N_0}, \quad \text{for } n \text{ near unity}$$

where, n is the index of refraction, N_0 is the number of molecules per unit volume at standard temperature and pressure ($i.e.$, conditions of index of refraction) , δ is the depolarization factor.

The optical depth can be related to the "thickness" of the atmosphere. If Z is the thickness in Km-amagats then:

$$\tau_{ray}(\lambda) = \int_0^z N(z) \cdot \sigma(\lambda) dz \quad \approx \quad \sigma(\lambda) \cdot \int_0^z N(z) \cdot dz \quad = \quad \mathrm{Z} \cdot N_0 \cdot \sigma(\lambda)$$

The index of refraction has a wavelength dependence. This is usually represented by two constants, A and B, as follows:

$$(n-1) = \mathrm{A} \cdot \left(1 + \frac{\mathrm{B}}{\lambda^2}\right), \quad \lambda \text{ in } \mu\text{m}$$

$$(n-1)^2 = \mathrm{A}^2 \cdot \left(1 + \frac{2\mathrm{B}}{\lambda^2} + \frac{\mathrm{B}^2}{\lambda^4}\right)$$

$$\tau_{ray} = \mathrm{Z}\frac{a_0}{\lambda^4}\left(1 + \frac{a_1}{\lambda^2} + \frac{a_2}{\lambda^4}\right) \quad \text{where,} \quad a_0 = \frac{32\pi^3}{3N_0^2} \cdot \frac{6+3\delta}{6-7\delta} \cdot \sum_{i=1}^{n} q_i \cdot \mathrm{A}^2,$$

$$a_1 = \sum_{i=1}^{n} q_i \cdot 2\mathrm{B}, \quad \text{and} \quad a_2 = \sum_{i=1}^{n} q_i \cdot \mathrm{B}^2$$

$$\tau_{ray}(H_2) = \mathrm{Z} \cdot \frac{2.19 \cdot 10^{-4}}{\lambda^4}\left(1 + \frac{0.0157248}{\lambda^2} + \frac{0.0001978}{\lambda^4}\right), \quad \lambda \text{ in } \mu m$$

The wavelength dependence in the VIAMP code is taken from Dalgarno and Williams (ApJ, 1962). This equation is similar to the equation above, however, it differs in the λ^8 term.

$$\sigma_{ray}(H_2) = \left(\frac{8.14 \cdot 10^{-13}}{\lambda^4} + \frac{1.28 \cdot 10^{-6}}{\lambda^6} + \frac{1.61}{\lambda^8}\right) \text{ cm}^2, \quad \lambda \text{ in } A$$

The optical depth per Km-amagat of hydrogen, denoted by $\tau_1(H_2)$, is then

$$\tau_1(H_2) = 2.687 \cdot 10^{24} \cdot \sigma_{ray}(H_2) = 2.687\left(\frac{8.14 \cdot 10^{11}}{\lambda^4} + \frac{1.28 \cdot 10^{18}}{\lambda^6} + \frac{1.61 \cdot 10^{24}}{\lambda^8}\right), \quad \lambda \text{ in } A$$

$$\tau_1(H_2) = 2.687\left(8.14 \cdot 10^{-21} f^4 + 1.28 \cdot 10^{-30} f^6 + 1.61 \cdot 10^{-40} f^8\right), \quad f = \frac{10^8}{\lambda} \quad \text{and} \quad \lambda \text{ in } A$$

And the total optical depth for a mixture of gases is given as

$$\tau_{ray} = \tau_1(H_2) \cdot \sum_i Z_i \cdot \frac{(n_i - 1)^2}{(n_{H_2} - 1)^2}$$

Note that this formulation always uses the wavelength dependence of hydrogen, even if other gases are used.

$$(n-1) = \mathrm{A} \cdot \left(1 + \frac{\mathrm{B}}{\lambda^2}\right), \quad \lambda \text{ in } \mu\text{m}$$

$$\text{ray} = \frac{(n-1)^2}{(n_{H_2}-1)^2}$$

from Allen (pg. 92)

	ray	n	A (10^-5)	B (10^-3)	alpha (10^-24)	delta	a0 um^4/Km
air	4.4459	1.0002918	28.71	5.67	3.40	0.031	10.7E-4
H2	1.0000	1.0001384	13.58	7.52	1.61	0.02	2.35E-4
HE	0.0641	1.0000350	3.48	2.30			
O2	3.8634	1.000272	26.63	5.07		0.054	
N2	4.6035	1.000297	29.06	7.7	3.44	0.030	10.9E-4
H2O	3.3690	1.000254					
CO2	10.5611	1.0004498	43.9	6.4		0.09	
CO	5.8247	1.000334	32.7	8.1			
NH3	7.3427	1.000375	37.0	12.0			
NO	4.6035	1.000297	28.9	7.4			
CH4	10.1509	1.000441					

The depth of penetration of Rayleigh scattering is computed for various objects using the data above. A summary of the characteristic of the plots in Figure 9 is given below (g is gravity in cm/s^2, u = molecular weight in gm/mole, Z = KM-amagats per Bar).

Earth P0 = 1013.25, g= 981, u=28.97, Z=7.88

wave(um):	0.1000	0.2000	0.2640	0.3000	0.4000	0.5000
tau(P0):	209.57	6.95	2.05	1.19	0.36	0.14
P(tau=1):	4.8	145.7	493.2	851.0	2834.7	7094.5

Jupiter P0 = 1000.00, g=2425.3, u=2.22, Z=37.45/0.9

wave(um):	0.1000	0.2000	0.2640	0.3000	0.4000	0.5000
tau(P0):	299.82	8.62	2.47	1.42	0.42	0.17
P(tau=1):	3.3	116.1	405.2	706.3	2390.8	6030.4

Saturn P0 = 1000.00, g= 1000, u = 2.14, Z=100.5/0.96

wave(um):	0.1000	0.2000	0.2640	0.3000	0.4000	0.5000
tau(P0):	754.31	21.68	6.21	3.56	1.05	0.42
P(tau=1):	1.3	46.1	161.0	280.7	950.3	2396.9

Titan P0 = 1500.00, g= 136, u= 28.0, Z=58.85

wave(um):	0.1000	0.2000	0.2640	0.3000	0.4000	0.5000
tau(P0):	5378.65	152.59	43.58	24.98	7.37	2.92
P(tau=1):	0.3	9.8	34.4	60.1	203.6	513.9

The gas continuum absorption contains a term which is supposed to represent the effect of Raman scattering due to the ν_1 vibration of H_2 at 4161 cm^{-1}. Ref.: Belton et al. (1971). Atm. of Uranus. ApJ. 164, 191-209

$$\tau_{raman} = 0.0208 \cdot \tau_{ray}$$

Rayleigh Scattering : Phase Function

Definitions of Angles

See Figure 11. The angle between the incident flux and the local normal is given by α and $\mu_0 \equiv \cos(\alpha)$. The angle between the local normal and the observer is given by ϵ and $\mu \equiv \cos(\epsilon)$. The phase angle, θ, is the angle between the incident and emission through the origin. The azimuthal angle, $\Delta\psi$, is the projection of the solar incidence and emission directions onto the local horizon. Using spherical trigonometry cosine laws (e.g., see CRC pg. 146) we can easily obtain the value of $\Delta\psi$) given $\mu_0, \mu, \text{and} \theta$. For solar scattering to an observer above the atmosphere the angles are related by:

$$\cos(\theta) = \sqrt{((1-\mu_0^2)(1-\mu^2))} \cdot \cos(\Delta\psi) \mp \mu\mu_0 \quad (\text{S} \; = \; +, \; \text{T} \; = \; -)$$

This is the method employed by Chandrasekar, Liou. and others. We could also write the equations in terms of the scattering angles. The scattering angle between incidence and emission, Θ, which is related to the phase function, is given by $\Theta = \pi - \theta$. The scattering angle in the horizontal plane, $\Delta\phi$ is related to the azimuthal angle, and is given by $\Delta\phi = \pi - \Delta\psi$. This definition is used by Hansen, Tomasko, and Danielson. It is the method employed within the VIAMP programs. Note the sign change between the two methods.

$$\cos(\Theta) = \sqrt{((1-\mu_0^2)(1-\mu^2))} \cdot \cos(\Delta\phi) \pm \mu\mu_0 \quad (\text{T} \; = \; +, \; \text{S} \; = \; -)$$

The Rayleigh phase function is given in terms of scattering angles or phase angles by

$$P_{ray}(\Theta) = \frac{3}{4}(1+\cos^2(\Theta)) = \frac{3}{4}(1+\cos^2(\theta)) = P_{ray}(\theta)$$

The $\cos^2$ term can be expressed in terms of μ, μ_0, and $\Delta\phi$ as

$$\cos^2(\Theta) = ((1-\mu_0^2)(1-\mu^2)) \cdot \cos^2(\Delta\phi) \pm 2\mu\mu_0\sqrt{((1-\mu_0^2)(1-\mu^2))} \cdot \cos(\Delta\phi) + \mu^2\mu_0^2$$

$$\cos(2\Delta\phi) = 2\cos^2(\phi) - 1 \quad \text{or,} \quad \cos^2(\phi) = \tfrac{1}{2}\cos(2\Delta\phi) + \tfrac{1}{2}$$

$$\cos^2(\Theta) = \frac{1}{2}((1-\mu_0^2)(1-\mu^2)) \cdot \cos(2\Delta\phi) \pm 2\mu\mu_0\sqrt{((1-\mu_0^2)(1-\mu^2))} \cdot \cos(\Delta\phi) + \mu^2\mu_0^2 + \frac{1}{2}((1-\mu_0^2)(1-\mu^2))$$

So the Rayleigh phase function can be written in terms of 3 Fourier moments:

$$P_{ray}(\Theta) = P_1 + P_2\cos(\Delta\phi) + P_3\cos(2\Delta\phi)$$

where the 3 coefficients are equal to

$$P_0 = \tfrac{3}{4}(1 + \mu^2\mu_0^2 + \tfrac{1}{2}((1-\mu_0^2)(1-\mu^2))) = \tfrac{3}{8}(3 + 3\mu^2\mu_0^2 - \mu_0^2 - \mu^2)$$

$$P_1 = \pm\tfrac{3}{2}\mu\mu_0\sqrt{((1-\mu_0^2)(1-\mu^2))} \quad (\text{T = +, S = -})$$

$$P_2 = \tfrac{3}{8}((1-\mu_0^2)(1-\mu^2))$$

Richardson Number

Dynamically significant stability indicator (similar to Burger number). As $Ri \rightarrow 0$ convection becomes strong.

$$\frac{Ri}{Ro} = \frac{\text{Effect of verticle motions}}{\text{Effect of Non} - \text{linear terms}}$$

when the Rossby Number, $Ro \geq 1$ then the static stability bouyancy frequency, $N = Ri/Ro$. When $Ro \ll 1, N = B_o/Ro$

$$R_i = \frac{g \cdot \frac{d\Theta}{dz}}{T_0 \cdot \left(\frac{dU}{dz}\right)^2} = \frac{N^2}{\left(\frac{dU}{dz}\right)^2}$$

Stone 1972 JAS **29** p. 405. For Jupiter: $d\Theta/dz = 6.0 \cdot 10^{-10}$ K/cm and $du/dz = 3.8 \cdot 10^{-4}$ cm/s, then $Ri = 3.7 \cdot 10^{-3}$

Rossby Number

A small Rossby number indicates that the geostrophic approximation is valid (*i.e.*, $du/dt = dv/dt = 0$) and that the Coriolis acceleration, $v \cdot f$, is greater than the horizontal fluid acceleration, du/dt.

$$Ro = \frac{\frac{du}{dt}}{vf} \approx \frac{v^2/r}{vf} = \frac{v/r}{f} = \frac{U}{f_0 \cdot L}$$

For the GRS, hurricanes, etc.

$$Ro = \frac{V_t a/b^2/\eta^3}{f}$$

$$\eta^2 = \frac{\cos^2(\theta) + (a^4/b^4) \cdot \sin^2(\theta)}{\cos^2(\theta) + (a^2/b^2) \cdot \sin^2(\theta)}$$

where V_t is the tangential velocity, a is the semimajor axis, b is the semiminor axis, and θ is the angle between the semi-major axis and the direction of motion. Mitchell *et al.* 1981 JGR **86** p.8751.

	terrestrial	Red Spot	FA/BC/DE	small.spots
Vt		110 m/s	120 m/s	
a		1.1E7 m	4.9E6 m	
Ro(0)		.36	.36	
Ro(90)		.04	.08	
g.lat		22 S	33 S	40.5 S

```
Great Red Spot
40000 x 13000 km  (1879-1882)
20-60 m/s velocity around spot, 6 day period
bounded by Jets at -19.5 (-60 m/s) and -27.0 (+50 m/s) graphic latitude
long drift = -3.4 m/s
+/- 1/2 degree longitude oscillation with 90 day period
anticyclonic (counterclockwise in southern hemisphere)
```

Saturation Vapor Pressure

Given T in Kelvin these equations will give e_s in milli-Bar.
from Fleagle and Businger, Vol.5, pg. 62 (QC880.F59)
The first law can be written as

$$L = T \cdot (S_2 - S1) = U_2 - U_1 + P_s \cdot (\alpha_2 - \alpha_1)$$

where $\alpha_1 \equiv 1/\rho_1$ and index 1 refers to the liquid phase and index 2 refers to the gas phase.
For an isothermal change of phase, the Clausius-Clapeyron equation has the form

$$\frac{dP_s}{dT} = \frac{L}{T \cdot (\alpha_2 - \alpha_1)}$$

Water vapor behaves like an ideal gas and $\alpha_2 >> \alpha_1$ for a change in state.

$L \approx 2.5 \cdot 10^3$ Joules/gm
$L \approx 2.824 \cdot 10^3$ Joules/gm over ice
$R_w = R_*/\overline{\mathrm{mw}}_w = 8.3143/18.016 = 0.4615$ Joules/gm/K
$P_s = \rho_2 \cdot R_w T$

$$\frac{dP_s}{dT} = \frac{L}{T \cdot (\alpha_2 - \alpha_1)} \simeq \frac{L \cdot \rho_2}{T} = \frac{L \cdot P_s}{R_w \cdot T^2}$$

$$d\log(e_s) = \frac{dP_s}{P_s} = \frac{L}{R_w} \cdot \frac{dT}{T^2}$$

$$\log_e e_s = \int_{T=T}^{T_0} \frac{L}{R_w} \cdot \frac{dT}{T^2} = \frac{-L}{R_w}|_T^{T_0} + C = \frac{-L}{R_w \cdot T_0} + \frac{L}{R_w \cdot T} + C$$

at triple point all 3 phases can exist in equilibrium, 0.0098° C and $P_s = 6.11$ mB

$$e_s(T = T_0) = 6.11$$

$L/R_w = 5417.12$
$L/(R_w * T_0) = 19.8313$
$6.11 \cdot \exp\left(L/(R_w * T_0)\right) = 2.504 \cdot 10^9$

$$e_s(T) = 6.11 \cdot \exp\left(5417\left(\frac{1}{T_0} - \frac{1}{T}\right)\right) = 2.504 \cdot 10^9 \cdot \exp\left(\frac{5417}{T}\right)$$

Undocumented fit is used in the program watsat.F (over liquid)

$$e_s = 2.229 \cdot 10^9 \cdot \exp\left(-5385/T\right)$$

Note this is the same equation as above, except that it assumes L =2485.2 Joules/gm and $T_0 = 273.15°$ K

Another undocumented fit is given (but not used) in the program watsat.F

$$e_s = 0.001 \cdot \exp\left(a/T + b + c\log(T) + d\cdot T + e\cdot T^2\right)$$

coef	over ice	over water
a	-5631.1206	-2313.0338
b	-8.363602	-164.03307
c	8.2312	38.053682
d	$-3.861449 \cdot 10^{-2}$	$-1.3844344 \cdot 10^{-1}$
e	$2.77494 \cdot 10^{-5}$	$7.4465367 \cdot 10^{-5}$

From Rogers and Yau, pg. 16

$$e_s = 6.112 \cdot \exp\left(\frac{a\cdot(T-273.16)}{T-b}\right)$$

coef	Rogers & Yau
coef	over water
a	17.67
b	29.66

Murray, F.W. 1966. "On the computation of Saturation Vapor Pressure" *J. Appl. Meteor.* **6** p.204

$$e_s = 6.1078 \cdot \exp\left(\frac{a\cdot(T-273.16)}{T-b}\right)$$

coef	Murray	Murray
coef	over ice	over water
a	21.8745584	17.2693882
b	7.66	35.86

Saucier, W.J.1883. "Principles of Meteorological Analysis" *Dover* pg. 9 who uses values of Tetens (1930). Note, he used $10^{(a'T/(T-b'))}$ with T in Centigrade so to convert into the form above $a = \log 10 \cdot a'$ and $b = 273.16 - b'$.

coef	over ice	over water
a	21.875	17.27
b	7.66	35.86

Thermal Wind Equation

Relationship between meridional temperature gradient and vertical wind shear due to non-uniform horizontal heating. Note: $dy = R_e \cdot d\varphi$. Hydrostatic and geostrophic balance yields:

$$\frac{\partial U_g}{\partial P} = \frac{1}{f}\frac{\partial}{\partial y}\left(\frac{1}{\rho}\right) = \frac{R_g}{f \cdot P}\frac{dT}{dy}$$

$$U(z_2) = U(z_1) - \frac{R}{f} \cdot \frac{dT}{dy}\bigg|_P \log(P_1/P_2)$$

$$f_0 \frac{dU}{dz} = \frac{R_g}{H}\frac{dT}{dy}$$

Velocity of sound

$$\frac{dU}{dt} + \frac{1}{\rho}\frac{\partial \rho}{\partial x} = 0$$

$$\frac{d\rho}{dt} + \rho \frac{\partial U}{\partial x} = 0$$

$$\frac{d \log(\Theta)}{dt} = 0$$

$$V_s = \sqrt{\gamma R_g T}, \quad \gamma = \frac{C_p}{C_v}$$

Vorticity

relative vorticity tends to be conserved in storm systems.

$$\zeta = \hat{k} \cdot \vec{\nabla} \times \vec{V} = \frac{dv}{dx} - \frac{du}{dy}$$

absolute vorticity tends to be conserved following the motion (*i.e.*, forecasting).

$$\eta \equiv \zeta + f = \hat{k} \cdot \vec{\nabla} \times \vec{V}_a$$

if η is positive then it is *cyclonic* and if η is negative then *anti-cyclonic*.

Weighting Functions

The outgoing radiance or upward radiance, R^u, for channel i from an atmospheric column is given by

$$R_i^u = \int_{p(z=0)}^{0} B_i(T(p)) \cdot \frac{\partial \tau_\nu^u(p)}{\partial \ln p} d \ln p$$

The *weighting function* is defined as

$$W(p) \equiv \frac{\partial \tau_\nu^u(p)}{\partial \ln p}$$

and the *contribution function* is the integrand of the outgoing radiance calculation

$$C(p) \equiv B_i(T(p)) \cdot \frac{\partial \tau_\nu^u(p)}{\partial \ln p}$$

and can be normalized as

$$C(p) \equiv \frac{B_i(T(p))}{R_i^u} \cdot \frac{\partial \tau_\nu^u(p)}{\partial \ln p}$$

the information content will require derivative w.r.t. temperature of the radiation transfer equation and, therefore, the *Kernel* function represents the best estimate of the sounding level for temperature retrievals.

$$K(p) \equiv \frac{\partial B_i(T(p))}{\partial T} \cdot \frac{\partial \tau_\nu^u(p)}{\partial \ln p}$$

Wind Measurement

The meridional velocity is the distance, $r \cdot \Delta\theta$, divided by the time difference, Δt. Since the radius is given in Km's and the time is given in hours we need factors of 1000 and 3600, respectively, to convert to meters per second. If θ is given in degrees then:

$$V = 1000R(\theta) \cdot \left(\frac{(\pi/180)(\theta_1 - \theta_0)}{3600\Delta t} \right)$$

To calculate the zonal velocity we need to calculate the displacement along a latitude circle, (i.e., $\Delta\lambda/360$), during the time interval Δt.

$$\Delta\lambda = \lambda(t_1) - \lambda(t_0)$$

$$U = 2\pi R(\overline{\theta}) \cos(\overline{\theta}) \cdot \frac{1000}{3600} \cdot \left(\frac{\Delta\lambda}{360\Delta t} \right)$$

Alternatively, we can calculate the period of rotation of the measured feature, P, and define the wind relative to the system of measure, P_{ref} as follows:

$$P = \frac{360\Delta t}{\Delta\lambda + \frac{360\Delta t}{P_{ref}}}$$

so that,

$$U = 2\pi R(\overline{\theta}) \cos(\overline{\theta}) \cdot \frac{1000}{3600} \cdot \left(\frac{1}{P} - \frac{1}{P_{ref}} \right)$$